**Correcting for radial distortion in spherical lenses**

**OVERVIEW**

Radial distortion is the approximately uniform spherical increase or decrease in magnification of an image which occurs in an image when viewed through a spherical lens (Chari & Veeraraghavan, 2014). When taking photographs of images using consumer digital cameras with spherical lenses, the dimensions and of the real-world object will be warped according to the object’s distance from the center of the camera’s field of view. Using the example of a gridding pattern, all real-world lines will appear to be either progressively stretched (pincushion; positive) or shrunken (barrel; negative) as the lines move away from the midpoint of an image captured with a spherical lens (*Figure 1*). Removing this distortion is essential in photogrammetry to ensure that measurements generated from scaled photographs are true to their real-world proportions.

1.  b)

**Figure 1: a)** Pincushion (positive) distortion pattern of a co-planar checkerboard pattern caused by spherical aberration; central squares are larger compared to the outer squares. **b)** Barrel (negative) distortion pattern of a co-planar checkerboard pattern caused by spherical aberration; central squares are smaller compared to the outer squares. Images from: (Chari & Veeraraghavan, 2014).

**METHOD**

This method will remove the progressive magnification errors with respect to *r* (distance from the image center) in a digital image (or landmarks generated from a digital image). It generates an estimated correction factor for each *x*- and *y*-component of an image’s distorted Cartesian coordinates in order to restore an object’s real-world proportions within the image. This method applies to both pincushion (positive) and barrel (negative) distortion patterns (*Figure 1*). Engineering solutions (camera and object levelling) will be used to minimize the effect of tangential distortion so that it can be approximated as ≈ 0. The following method is adapted from (Mikhail et al., 2004).

**Determining the distortion correction equation**

This step is the “calibration” step where the correction factor for any object at a *single* specific focal length using a *single* specific camera lens/make/model is generated. A checkerboard pattern which covers the area where the object will be photographed is set up at a fixed distance from the camera. This distance will be the same as the distance used to photograph the object so it is imperative that the entire object be visible in the field of view at the specific focal distance. The center of the checkerboard must be aligned orthogonally along the central axis of the camera’s field of view such that tangential (or decentralizing) distortion is minimised.

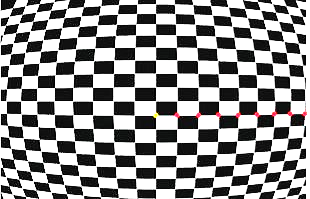
1. **Capture the calibration image(s):** Using the camera’s “cross” marking at the center of the image preview screen, align the checkerboard midpoint with the cross and ensure that the horizontal and vertical image axes remain orthogonal to each other. Capture multiple images if necessary and import the best one(s) into the StereoMorph landmarking software. *Do not* adjust the position of the image’s horizontal, vertical axes or midpoint (such as by “cropping” or rotation) because they alter the position of the true midpoint and axes in the following steps. Use only the “best aligned” picture(s) in the following steps.
2. **Landmark the control line:** Place landmarks on the control line of the image so that the control line spans all of the way from the image midpoint to the edge of the image’s *longest* dimension. A minimum of 5-7 landmarks should be positioned at checkerboard corners crossing the control line to ensure an adequate number of data points for the regression step.
3. **Set image midpoint:** Midpoint of the image will be approximated as the center of the image; this corresponds to the point located at the halfway-point of the image’s *x*-axis and the halfway-point of the image’s *y*-axis. Apply the appropriate translational transformations to the Cartesian coordinate datasets such that all of the coordinates are centered around the image’s midpoint.
4. **Find radial distortion regression coefficients:** The radial distortion coefficients can be determined by taking a series of coordinates on a straight line (control line) which passes through the image midpoint (yellow point) and running a least squares regression for a curvilinear function on the distances between the points as multiples of the real-world radius (*r’*). The control line (blue) is broken up into segments which align with the warped grid in order to establish the landmarks of the control line. A least squares regression is used to fit the parameters of equation [1], an odd power polynomial function whose terms knr2n+1 are derived from lens theory (Mikhail et al., 2004). The U.S. Geological Survey (USGS) collimator cross calibration method uses a version of equation [1] containing terms up to the 7th power (equation [2]) which will also be the model used in this method (Mikhail et al., 2004). The warped radius (*r*), as it appears on the distorted image, is the Pythagorean (Euclidean) distance from the midpoint (*x*m, *y*m), as described in equation [3].

[1]

[2]

[3]

*Figure 2* shows the relationship between the image length *r* and the multiples of *r’*.

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***r***

***2r***

***3r***

**Figure 2.** Relationship between the observed radius (*r*) and multiples of the original radius (*r’*). Because this control line passes horizontally through the image’s midpoint, the radial distortion was derived from only the *x*-component of the image.

1. **Correct landmark and curve coordinates:** After all object images have been captured and landmarked, individual *x*- and *y*-components of the landmark and curve coordinates are corrected separately by applying formulas [4a] and [4.b], respectively. *x* and *y* are the components of the radius on the distorted image and *x’* and *y’* are the corrected components.

[4.a]

[4.b]

**WORKS CITED**

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